Traffic Flow Dynamics
Traffic Flow Dynamics

Data, Models and Simulation

Translated by Martin Treiber and Christian Thiemann

Springer
Preface

In order to keep people moving in times of rising traffic and limited resources, science is challenged to find intelligent solutions. Over the past few years, contributions from engineers, physicists, mathematicians, and behavioral psychologists have lead to a better understanding of driver behavior and vehicular traffic flow. This interdisciplinary field will surely produce further advances in the future. The focus is on new applications ranging from novel driver-assistance systems, to intelligent approaches to optimizing traffic flow, to the precise detection of traffic jams and the short-term forecasting of traffic for dynamic navigation aids.

This textbook offers a comprehensive and didactic account of the different aspects of vehicular traffic flow dynamics and how to describe and simulate them with mathematical models. We hope to make this fascinating field accessible to a broader readership; to date, it has only been documented in specialized scientific papers and monographs.

Part I describes how to obtain and interpret traffic flow data, the basis of any quantitative modeling. The second and main part is devoted to the different approaches and models used to mathematically describe traffic flow. The starting point of most models are the basic concepts of physics—many-particle systems, hydrodynamics, and classical Newtonian mechanics—augmented by behavioral aspects and traffic rules. At the website1 accompanying this book, the reader can interactively run a selection of traffic models and reproduce some of the simulation results displayed in the figures. Part III gives an overview of major applications including traffic-state estimation, fuel consumption, and emission modeling, determining travel times (the basis of dynamic navigation), and how to optimize traffic flow.

The book is written for students, lecturers, and professionals of engineering and transportation sciences and for interested students in general. It also offers material for project work in programming, numerical methods, simulation, and mathematical modeling at college and university level. The reference implementations in the

1 see: www.traffic-flow-dynamics.org
multi-model open-source vehicular traffic simulator *MovSim*\(^2\) can be used as a starting point for the reader’s own simulation experiments and model development.

This work originates from the lecture notes of courses in traffic flow dynamics and modeling at the Dresden University of Technology, Germany; these have been previously published, by the same publisher, in the German book “Verkehrsdy-namik und Simulation”. The English edition has been updated and significantly extended to include new topics, e.g., on model calibration. To underline its textbook character, it contains many problems with elaborated solutions.

We thank all colleagues at our Department for Traffic Econometrics and Modeling at the Dresden University of Technology, particularly Dirk Helbing, for various scientific discussions and stimulations. We would also like to thank Marietta Seifert, Christian Thiemann, and Stefan Lämmer for suggestions and corrections. Special thanks go to Martin Budden for reviewing the manuscript as a native English speaker. He is also one of the main contributors to *MovSim*. Finally, we would like to thank Martina Seifert, Christine and Hanskarl Treiber, Ingrid, Bernd, and Dörte Kesting, Claudia Perlitus, and Ralph Germ who contributed to the book with valuable suggestions.

Dresden, June 2012

Martin Treiber
Arne Kesting

---

\(^2\) see: [www.movsim.org](http://www.movsim.org)
Contents

1 Introduction .................................................................................. 1

Part I Traffic Data

2 Trajectory and Floating-Car Data .................................................... 7
  2.1 Data Collection Methods .............................................................. 7
  2.2 Time-Space Diagrams ................................................................. 9
  Problems ....................................................................................... 10

3 Cross-Sectional Data ................................................................. 13
  3.1 Microscopic Measurement: Single-Vehicle Data ......................... 13
  3.2 Aggregated Data ....................................................................... 15
  3.3 Estimating Spatial Quantities from Cross-Sectional Data ............. 17
    3.3.1 Traffic Density ................................................................. 17
    3.3.2 Space Mean Speed ........................................................ 21
  3.4 Determining Speed from Single-Loop Detectors ......................... 22
  Problems ....................................................................................... 23

4 Representation of Cross-Sectional Data ............................................. 25
  4.1 Time Series of Macroscopic Quantities .................................... 25
  4.2 Speed-Density Relation ............................................................ 27
  4.3 Distribution of Time Gaps .......................................................... 30
  4.4 Flow-Density Diagram ............................................................. 31
  4.5 Speed-Flow Diagram .................................................................. 35
  Problems ....................................................................................... 35

5 Spatiotemporal Reconstruction of the Traffic State ....................... 37
  5.1 Spatiotemporal Interpolation ....................................................... 37
  5.2 Adaptive Smoothing Method ....................................................... 40
| 5.2.1 | Characteristic Propagation Velocities | 41 |
| 5.2.2 | Nonlinear Adaptive Speed Filter | 42 |
| 5.2.3 | Parameters | 43 |
| 5.2.4 | Testing the Predictive Power: Validation | 43 |
| 5.2.5 | Testing the Robustness: Sensitivity Analysis | 44 |
| 5.3 | Data Fusion | 45 |
| 5.3.1 | Model-Based Validation of a Data Fusion Procedure | 47 |
| 5.3.2 | Weighting the Data Sources | 48 |

Problems | 50 |

### Part II Traffic Flow Modeling

| 6 | General Aspects | 55 |
| 6.1 | History and Scope of Traffic Flow Theory | 55 |
| 6.2 | Model Classification | 56 |
| 6.2.1 | Aggregation Level | 56 |
| 6.2.2 | Mathematical Structure | 59 |
| 6.2.3 | Other Criteria | 61 |
| 6.3 | Non-Motorized Traffic | 63 |

Problems | 65 |

### 7 Continuity Equation

| 7.1 | Traffic Density and Hydrodynamic Flow-Density Relation | 67 |
| 7.2 | Continuity Equations for Several Road Profiles | 69 |
| 7.2.1 | Homogeneous Road Section | 70 |
| 7.2.2 | Sections with On- and Off-Ramps | 71 |
| 7.2.3 | Changes in the Number of Lanes | 72 |
| 7.2.4 | Discussion | 74 |
| 7.3 | Continuity Equation from the Driver’s Perspective | 75 |
| 7.4 | Lagrangian Description | 77 |

Problems | 79 |

### 8 The Lighthill–Whitham–Richards Model

| 8.1 | Model Equations | 81 |
| 8.2 | Propagation of Density Variations | 83 |
| 8.3 | Shock Waves | 84 |
| 8.3.1 | Formation | 84 |
| 8.3.2 | Derivation of the Propagation Velocity | 86 |
| 8.3.3 | Vehicle Speed Versus Propagation Velocities | 87 |
| 8.4 | Numerical Solution | 90 |
| 8.5 | LWR Models with Triangular Fundamental Diagram | 91 |
| 8.5.1 | Model Parameters | 92 |
| 8.5.2 | Characteristic Properties | 93 |
8.5.3 Model Formulation with Measurable Quantities . . . . . . 96
8.5.4 Relation to Car-Following Models . . . . . . . . . . . . . . 97
8.5.5 Definition of Road Sections . . . . . . . . . . . . . . . . . . 99
8.5.6 Modeling Bottlenecks . . . . . . . . . . . . . . . . . . . . . . 100
8.5.7 Numerical Solution of the Cell-Transmission
Model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 105
8.5.8 Solving the Section-Based Model . . . . . . . . . . . . . . 108
8.5.9 Examples . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 113
8.6 Diffusion and Burgers’ Equation . . . . . . . . . . . . . . . . . . . 121
Problems . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 123
9 Macroscopic Models with Dynamic Velocity . . . . . . . . . . . . 127
9.1 Macroscopic Acceleration Function . . . . . . . . . . . . . . . . . 127
9.2 Properties of the Acceleration Function . . . . . . . . . . . . . . 130
9.2.1 Steady-State Flow . . . . . . . . . . . . . . . . . . . . . . . . . . 130
9.2.2 Plausibility Conditions . . . . . . . . . . . . . . . . . . . . . . . 130
9.3 General Form of the Model Equations . . . . . . . . . . . . . . . 132
9.3.1 Local Speed Adaptation . . . . . . . . . . . . . . . . . . . . . . 132
9.3.2 Nonlocal Anticipation . . . . . . . . . . . . . . . . . . . . . . . 133
9.3.3 Limiting Case of Zero Adaptation Time . . . . . . . . . . 133
9.3.4 Pressure Term . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 134
9.3.5 Diffusion Terms . . . . . . . . . . . . . . . . . . . . . . . . . . . . 136
9.3.6 On- and Off-Ramp Terms . . . . . . . . . . . . . . . . . . . . 137
9.4 Overview of Second-Order Models . . . . . . . . . . . . . . . . . . 137
9.4.1 Payne’s Model . . . . . . . . . . . . . . . . . . . . . . . . . . . . 138
9.4.2 Kerner–Konhäuser Model . . . . . . . . . . . . . . . . . . . . 140
9.4.3 Gas-Kinetic-Based Traffic Model . . . . . . . . . . . . . . . 142
9.5 Numerical Solution . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 145
9.5.1 Overview . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 145
9.5.2 Upwind and McCormack Scheme . . . . . . . . . . . . . . . 147
9.5.3 Approximating Nonlocalities . . . . . . . . . . . . . . . . . . 148
9.5.4 Criteria for Selecting a Numerical
Integration Scheme . . . . . . . . . . . . . . . . . . . . . . . . . . 148
9.5.5 Numerical Instabilities . . . . . . . . . . . . . . . . . . . . . . . 150
9.5.6 Numerical Diffusion . . . . . . . . . . . . . . . . . . . . . . . . 153
Problems . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 153
10 Elementary Car-Following Models . . . . . . . . . . . . . . . . . . 157
10.1 General Remarks . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 157
10.2 Mathematical Description . . . . . . . . . . . . . . . . . . . . . . . 159
10.3 Steady State Equilibrium and the Fundamental Diagram . . . 162
10.4 Heterogeneous Traffic . . . . . . . . . . . . . . . . . . . . . . . . . . . 164
10.5 Fact Sheet of Dynamical Model Characteristics . . . . . . . . 165
10.5.1 Highway Scenario . . . . . . . . . . . . . . . . . . . . . . . . . . 165
10.5.2 City Scenario . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 168
10.6 Optimal Velocity Model ........................................ 168
10.7 Full Velocity Difference Model ................................. 171
10.8 Newell’s Car-Following Model ................................. 173
Problems .......................................................... 178

11 Car-Following Models Based on Driving Strategies ............ 181
11.1 Model Criteria .................................................. 181
11.2 Gipps’ Model .................................................... 183
  11.2.1 Safe Speed ................................................. 183
  11.2.2 Model Equation .......................................... 184
  11.2.3 Steady-State Equilibrium ................................. 185
  11.2.4 Model Characteristics .................................. 185
11.3 Intelligent Driver Model ........................................ 187
  11.3.1 Required Model Properties .............................. 188
  11.3.2 Mathematical Description ............................... 188
  11.3.3 Parameters ............................................... 189
  11.3.4 Intelligent Braking Strategy ............................ 191
  11.3.5 Dynamical Properties ................................ 193
  11.3.6 Steady-State Equilibrium ............................... 195
  11.3.7 Improved Acceleration Function ......................... 196
  11.3.8 Model for Adaptive Cruise Control .................... 198
Problems .......................................................... 202

12 Modeling Human Aspects of Driving Behavior .................... 205
12.1 Man Versus Machine .......................................... 205
12.2 Reaction Times ............................................... 207
12.3 Estimation Errors and Imperfect Driving Capabilities ....... 210
  12.3.1 Modeling Estimation Errors ............................ 210
  12.3.2 Modeling Imperfect Driving ........................... 213
12.4 Temporal Anticipation ........................................ 214
12.5 Multi-Vehicle Anticipation .................................. 215
12.6 Brake Lights and Further Exogenous Factors ................ 218
12.7 Local Traffic Context ....................................... 219
12.8 Action Points .................................................. 220
12.9 The Wiedemann Car-Following Model ......................... 221
Problems .......................................................... 223

13 Cellular Automata .................................................. 225
13.1 General Remarks ............................................... 225
13.2 Nagel-Schreckenberg Model .................................. 229
13.3 Refined Models ................................................ 232
  13.3.1 Barlovic Model ............................................ 232
  13.3.2 KKW Model ............................................... 233
16.4 Validation ........................................ 333
Problems ............................................. 337

17 The Phase Diagram of Congested Traffic States ................. 339
17.1 From Ring Roads to Open Systems ......................... 339
17.2 Analysis of Traffic Patterns: Dynamic Phase Diagram .... 340
  17.2.1 Stability Class 1 .............................. 342
  17.2.2 Stability Class 2 .............................. 345
  17.2.3 Stability Class 3 .............................. 346
17.3 Simulating Congested Traffic Patterns
  and the Phase Diagram ............................. 347
17.4 Reality Check: Observed Patterns of Traffic Jams ........... 350
Problems ............................................. 350

Part III Applications of Traffic Flow Theory

18 Traffic Flow Breakdown and Traffic-State Recognition .... 355
  18.1 Traffic Flow Breakdown: Three Ingredients
to Make a Traffic Jam .............................. 355
  18.2 Do Phantom Traffic Jams Exist? ..................... 360
  18.3 Stylized Facts of Congested Traffic .................. 361
  18.4 Empirical Reality: Complex Patterns ................ 363
  18.5 Fundamentals of Traffic State Estimation ............. 364
Problems ............................................. 365

19 Travel Time Estimation ................................ 367
  19.1 Definitions of Travel Time ......................... 367
  19.2 The Method of Trajectories ......................... 368
  19.3 The Method of Accumulated Vehicle Counts .......... 369
  19.4 A Hybrid Method ................................ 371
  19.5 Virtual Stationary Detectors ....................... 373
  19.6 Virtual Trajectories ................................ 373
  19.7 Instantaneous Travel Time ......................... 375
Problems ............................................. 376

20 Fuel Consumption and Emissions ............................ 379
  20.1 Overview ........................................ 379
  20.1.1 Macroscopic Models ............................ 380
  20.1.2 Microscopic Models ............................ 382
  20.1.3 Relation Between Fuel Consumption
         and CO₂ Emissions .............................. 383
## Contents

20.2 Speed-Profile Emission Models .......................... 383  
20.3 Modal Emission Models .................................. 385  
20.3.1 General Remarks ...................................... 385  
20.3.2 Phenomenological Models ............................. 386  
20.3.3 Load-Based Models .................................... 387  
20.4 Physics-Based Modal Consumption Model ................. 388  
20.4.1 Driving Resistance ..................................... 388  
20.4.2 Engine Power. .......................................... 390  
20.4.3 Consumption Rate ..................................... 391  
20.4.4 Characteristic Map for Engine Efficiency .......... 392  
20.4.5 Output Quantities .................................... 394  
20.4.6 Aggregation to a Macroscopic Modal  
Consumption Model ........................................... 397  
Problems .......................................................... 397  

21 Model-Based Traffic Flow Optimization ................. 403  
21.1 Basic Principles .......................................... 403  
21.2 Speed Limits ............................................. 405  
21.3 Ramp Metering ............................................ 407  
21.4 Dynamic Routing ......................................... 411  
21.5 Efficient Driving Behavior and Adaptive Cruise Control. 412  
21.6 Further Local Traffic Regulations ....................... 416  
21.7 Objective Functions for Traffic Flow Optimization . 417  
21.7.1 Setting up the Frame. ................................. 417  
21.7.2 Constraining Conditions ............................. 418  
21.7.3 Examples ............................................. 419  

Solutions to the Problems ..................................... 423  

Index ................................................................. 495